DYNAMICAL GENERATION OF LIGHT FERMIONS

PANKAJ JAIN *

Physics Department, I.I.T., Kanpur, India 208016

Abstract

We show that composite fermions with masses much smaller than the scale of confinement arise naturally in certain models which admit dynamical breakdown of chiral symmetry. The models are such that to leading order some of the fermions remain massless but pick up small dynamical masses at subleading order.

1. Introduction

The existence of fermions with masses much smaller than the scale of electroweak symmetry breaking might be the result of an approximate chiral symmetry of the underlying model. There exist several composite models [1] based on this idea, but in most cases in the absence of fundamental scalars some of the fermions remain exactly massless and remaining fermions pick up masses of the order of the scale of confinement of the underlying strong dynamics, which may be equal to or larger than the scale of electro-weak symmetry breaking. Generation of nonvanishing masses for light fermions in a dynamical framework has proven to be a very difficult problem in all of the popular scenarios including technicolor [2], top condensate [3] as well as models in which fermions and/or electroweak bosons are composite [1]. In the present paper we display some situations in which some of the fermions dynamically acquire very small but non zero masses.

2. Light fermions in a large N chiral model

Dynamical light fermions can arise if the fermion representation is such that to leading order fermion condensate is prevented from forming. We have in mind some nonabelian gauge group and by leading order we mean leading order in either the loop expansion or the $1/N$ expansion [4] or a small gauge coupling parameter. In the present section we will confine ourselves to the cases in which $1/N$ is the small

*e-mail: pkjain@iitk.ernet.in

parameter and will discuss the generalizations in the later sections. The possibility of a systematic loop expansion within a dynamical framework is very interesting although there is no evidence that this is a reliable expansion scheme.

To give a simple example we consider a $SU(N)_1 \times SU(N)_2 \times SU(2)_1 \times SU(2)_2$ model with N large. The small parameter in this example is $1/N$. We introduce fermions in following representation of the $SU(N)_1 \times SU(N)_2 \times SU(2)_1 \times SU(2)_2$ gauge group.

$$a_L \rightarrow (N,1,1,1), \quad a_R \rightarrow (1,N,1,1)$$

$$b_L \rightarrow (N,1,1,1), \quad b_R \rightarrow (1,N,1,1)$$

$$c_L \rightarrow (N^*,1,2,1), \quad c_R \rightarrow (1,N^*,1,2)$$

The representation has been chosen such that the interaction is free of anomalies. We have not specified the transformation of these fermion representations under electroweak and color interactions. Generalization of the present model to include electroweak and color interactions will be discussed in a separate publication. The two SU(2) groups are assumed to be strong which will prevent the condensation of the fermions a or b with the fermion c. The strong SU(2) groups continue to have asymptotic freedom as long as $N < 11$. As we discuss later this model is expected to be confining because of its non-abelian nature and the gauge symmetry breaking is small. We assume the scale of confinement of the two SU(N) groups to be $\Lambda_{\rm conf}$.

The reason we are attracted to such $SU(N)_1 \times SU(N)_2$ type models is that the scale of chiral symmetry breaking will be considerably suppressed compared to the scale of confinement. Because of the presence of the strong SU(2) interactions, the fermion a_L can only condense with either a_R or b_R. We assume that it condenses only with one of these two right handed particles and the horizontal symmetry which rotates a into b remains unbroken. Based on our experience with QCD we expect this to be true. In any case we can always assign different electric charges to these two fermions to assure that a_L condenses only with a_R. To analyze the pattern of mass generation we note that all possible diagrams that can convert a left handed fermion to a right handed fermion contain atleast one internal fermion loop and are therefore suppressed by one power of N. All of these graphs will therefore vanish as $N \rightarrow \infty$ and chiral symmetry will remain unbroken. In arriving at this conclusion we have assumed that the dynamically generated fermion mass decreases as the value of the effective coupling decreases. Based on model calculations this is generally expected to be the case for QCD type vectorial theories. For example, the one loop analysis of Schwinger-Dyson shows that as α_s, treated as a constant, decreases the dynamically generated mass also decreases and eventually vanishes as the coupling goes below a critical value [5]. If instead a renormalization group improved expression is used for the effective coupling, such that the coupling continues to rise with decrease in momentum, then there is no critical point but the dynamical mass continues to decrease with the decrease in the value of effective coupling at some scale. We will assume this also to be the case for the present model. The Schwinger-Dyson (SD) equation including only the leading order contribution in the loop expansion of CJT effective action [6], which can lead to chiral symmetry breaking, is shown is Fig. 1.

To see how the $1/N$ suppression factor effects the scale of chiral symmetry breaking we use a simple model for the nonabelian gauge coupling. We assume that the coupling has the form

$$\alpha \approx 1/\log\left(q^2/\Lambda^2_{\rm conf}+1\right)$$

The reason for this choice [7] is simply that it interpolates between the correct asymptotic behavior and the popular infrared behavior [8] since it leads to $1/q^4$ momentum dependence for the hypergluon propagator. We take the scale at which this coupling becomes equal to 1 to be the scale of chiral symmetry breaking. For the chosen behavior the coupling becomes equal to 1 at $q^2/\Lambda^2_{\rm conf}=1.7$. This is roughly the scale of chiral symmetry breaking if the theory is vectorial. However in the present case the effective coupling is a factor of N smaller. To get the scale of chiral symmetry breaking in this case we set $\alpha/N=1$ to get $q^2/\Lambda^2_{\rm conf}=\exp(1/N)-1$. For N of the order of 10 this yields $q^2/\Lambda^2_{\rm conf}=0.1$, which shows a significant suppression factor. This model calculation atleast shows that for N large enough the scale of chiral symmetry is much smaller than the scale of confinement. We point out that if N is arbitrarily large but not infinite than in the model discussed above chiral symmetry breaking will take place. It is not clear, however, if this is true in reality because of our lack of knowledge of the behavior of nonabelian theories at low energies and the effective coupling may not increase monotonically with decrease in momentum. In any event there may exist a large range of values of N for which the chiral symmetry still takes place. We assume this to be the case.

This theory will behave very different from QCD in which case the scale of confinement is roughly the same as the scale of chiral symmetry breaking. In particular, in the present case to leading order we may simply ignore chiral symmetry breaking. Indeed in the limit as N$\rightarrow\infty$, there is no breakdown of chiral symmetry. This implies that to leading order all fermions will be massless. This conclusion holds not only for the elementary confined fermions but also for the composite fermions which are bound states of the type N$\times$N$\times$N$\times$..... . In the left-right theory under consideration there will infact exist several such states, the simplest one being made of N left handed or N right handed fermions. More complicated composite fermions may also be formed by including one or more hypergluons along with the N fermions. In the infinite N limit these are the only type of fermions allowed. One cannot, for example, have a bound state containing some left handed and some right handed fermions since there is no binding between left and right handed fermions in this limit.

We next consider the finite N corrections which will link the two composite fermions discussed above. In order to convert a left handed composite fermion to a right handed composite fermion we need to convert all the N fundamental left handed fermions into right handed fermions. This coupling of left and right handed composite fermions is shown in figure 2. Coversion of each of these fundamental left handed fermion into a right handed fermion is suppressed by one power of the ratio of the scale of chiral symmetry breaking and the confinement scale. We call this suppression factor ξ. The conversion of N elementary left handed fermions into right handed fermions will therefore be suppressed by ξ^N. This shows that the mass of these composite particles will be extremely small. The logic used above to get this suppression factor is very different from the usual intuition one has about bound states. However even the usual intuition applied to the present case shows that the suppression factor has to be very large. We are forming very tightly bound states of fermions which have mass much smaller than the scale of confinement. The binding energy is necessarily very large, and results in very small bound state mass. This argument, however, does not tell us whether the lightest state is a fermion or a boson. The systematic large N expansion gives a very good indication that the fermion has to be the lightest state. The boson states, which have a group theoretic structure $\mathrm{N}\times\bar{\mathrm{N}}$, are much heavier since they do not have the suppression factor ξ^N. The mass of these states in the large N limit is independent of N and therefore these states will have masses much smaller than the confinement scale but not as small as the fermion masses.

We note that because of finite N corrections the gauge symmetry $\mathrm{SU(N)}_1\times\mathrm{SU(N)}_2$ will be broken to $\mathrm{SU(N)}_{hc}$, where hc stands for hypercolor. However since this breaking is subleading the massive gauge particles will also be very light. Furthermore since the breaking is negligible to leading order, the theory is confining and all the physical states must be singlets under both $\mathrm{SU(N)}_1$ and under $\mathrm{SU(N)}_2$.

The model described in this paper naturally generates very tightly bound composite fermions which are much lighter than the scale of confinement. Generalizations of this model to include color and electroweak interactions are currently under consideration and will be described in a separate publication.

3. Acknowledgement

I would like to thank Chris Hill, Doug McKay, Herman Munczek, John Ralston, Joseph Schechter and especially Kimball Milton for useful discussions. This work was supported in part by the U.S. DOE contract no. DE-FG05-91ER-40636.

REFERENCES

[1] For a review of composite models see, I.A. D' Souza, C.S. Kalman, *Preons: Models of Leptons, Quarks and Gauge Bosons as Composite Objects* World Scientific, Singapore (1992).

[2] Review articles on technicolor include, E. Farhi and L. Susskind, Phys. Rep. **74**, 277 (1981) ; *Dynamical Gauge Symmetry Breaking* edited by E. Farhi and R. Jackiw (World Scientific, 1982); R. K. Kaul, Rev. Mod. Phys. **55**, 449 (1983); K. Lane, *An Introduction to Technicolor*, Lectures given at Theoretical Advanced Study Institute (TASI 93) in Elementary Particle Physics: The Building Blocks of Creation - From Microfermius to Megaparsecs, Boulder, CO, 6 Jun - 2 Jul 1993.

[3] Y. Nambu, in *New Theories in Physics*, proceedings of the XI International Symposium on Elementary Patricle Physics, Kazimierz, Poland, 1988, edited by Z. Ajduk, S. Pokorski and A. Trautman (World Scientific, Singapore, 1989); V. A. Miransky, M. Tanabashi and K. Yamawaki, Mod. Phys. Lett. **A4**, 1043 (1989); W. A. Bardeen, C. T. Hill and M. Lindner, Phys. Rev. **D41**, 1647 (1990); C. T. Hill, Phys. Lett. **266B**, 419 (1991); S. F. King, Phys. Rev. **D45**, 990 (1992); D.E. Clague, G.G. Ross, Nucl. Phys. B**364**, 43 (1991); S. P. Martin, Phys. Rev. **D46**, 2197 (1992).

[4] G. 't Hooft, Nucl. Phys. **B 72**, 461 (1974); E. Witten, Nucl. Phys. **B 160**, 57 (1979).

[5] V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theory* (World Scientific, Singapore, 1993); K. Higashijima, Prog. Theor. Phys. Suppl. **104**, 1 (1991); C. D. Roberts and A G. Williams, in *Progress in Particle and Nuclear Physics* **33**, 477 (1994) (Pergamon, Great Britain, 1994).

[6] J. Cornwall, R. Jackiw and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[7] J. L. Richardson, Phys.Lett. **82B**, 272 (1979).

[8] H. Pagels; S. Mandelstam, Phys. Rev. **D20**, 3223 (1979); N. Brown and M. R. Pennington, Phys. Rev. **D38**, 2266 (1988); **39**, 2723 (1989); U. Bar-Gadda, Nucl. Phys. **B163**, 312 (1980); M. Baker, J. S. Ball and F. Zachariasen, Nucl. Phys. **B186**, 531 (1981); **B186**, 560 (1981).